Effect of Spanwise Variation of Gust Velocity on Airplane Response to Turbulence

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When predicting the vertical response of a flexible aircraft to atmospheric turbulence, the designer may use either the assumption of flight through cylindrical turbulent waves, or through an isotropic field of random vertical flow, which is no longer uniform in span. In this first part, the paper is devoted to the demonstation of a very simple formula which makes it possible to determine the range of validity of both methods. It appears that the assumption of turbulence uniform in span gives a good approximation in the range of frequency associated with rigid-airplane motion. It is no longer valid, in general, at flexible modes natural frequencies. The method proposed by ONERA to compute transfer functions to isotropic turbulence, using lifting surface theory, is briefly described. However, this second part of the paper is mainly concerned with discussions of the power spectral densities of the response at different locations of the structure of the Concorde SST. These evaluations have been obtained both for isotropic turbulence and for turbulence uniform in span. The first method gives loads about 14% lower. In the third part, comparison is extended to fatigue damage, and consequence on fatigue life is emphasized.

Nomenclature

b	= spanwise dimension of the aircraft
$\tilde{F}(t)$	= inverse Fourier Transform of $F(\omega)$
$g(r)$	= transverse correlation function of turbulence
$G(\omega)$	= turbulence spectral model
$j_\lambda(x)$	= bessel function of first kind and order λ
$K(M,M',\omega)$	= kernel of the integral equation of the lifting surface theory
$K_v(x)$	= modified Bessel function of second kind and order v
l	= transverse coherence length of turbulence
L	= scale of turbulence
N_o	= mean number of zero crossings of turbulence per second
N_{os}	= mean number of zero crossings of output per second
$N_r(s)$	= number of cycles to failure under stress s
$p(M,t)$	= pressure field on the wing
$q_k(t)$	= generalized coordinate associated with mode k
r	= distance between two points
$R_w(M,M',\tau)$	= cross correlation of the vertical component of turbulence
s	= reference stress
$S_p(M,M',\omega)$	= cross power of the pressure field on the wing
$S_w(M,M',\omega)$	= cross power spectrum of the vertical component of turbulence
t	= time
T_f	= predicted fatigue life of the structure
$T_{ij}(\omega)$	= transfer function of mode i to excitation on mode j
V_o	= mean velocity of the aircraft
x,y,z	= orthogonal coordinate system, body axis
X^*	= complex conjugate of X
$\Gamma(x)$	= gamma function
$\Delta(\sigma_w)$	= probability density of σ_w
σ_s	= rms of the output
σ_w	= rms Gaussian patch of turbulence
τ	= time lag between two events
$\psi_k(M)$	= natural mode k of the structure
ω	= circular frequency

Introduction

BECAUSE of a drastic increase in size and flight speed, most of the new transport aircraft must be analysed very carefully from the point of view of their response to atmospheric turbulence.[1] The general harmonic analysis, based on the concepts proposed by Clementson[2] and Press,[3] is more often preferred to the previous approach, which considered atmospheric turbulence as a random set of discrete gusts. Responses have been predicted most commonly with the assumption that the vertical component of the turbulence velocity varies along the flight path, but does not vary along the span. This assumption is attractive because the associated computations are less numerous than those for the more general case of isotropic turbulence. It has provided useful results for many aircraft, but prediction for larger aircraft may not be sufficiently accurate.

This assumption is discussed by comparing the transverse coherence length of the turbulent field to the spanwise dimension of the aircraft. It is clear that, if turbulence is nearly constant in span, the correlation factor between the opposite tips of the wings will be close to unity, and the transverse coherence length will be much greater than the spanwise dimension. Mathematical manipulation involving Bessel integrals makes it possible to determine this coherence length and, as a consequence, to decide whether the assumption of uniformity is valid or not. It is shown that the hypothesis gives a good working tool in the frequency range associated with rigid-airplane motion, but can no longer stand once flexible modes are concerned.

Following this remark, the prediction of the responses of a large flexible aircraft must be achieved taking into account the isotropy of the turbulent field. Following the work of Lin,[4] Fuller,[5] Diederich[6] and Eichenbaum,[7] a general method based on the use of the lifting surface theory has been derived at ONERA.[8] This method makes it possible to compute the cross power spectrum of the pressure field induced on a wing of arbitrary planform by isotropic turbulence. Following this technique, the response power spectral densities of the Concorde SST have been investigated and compared with the results of previous computations for which the turbulent field had been assumed uniform in span. This cross-checking makes it clear that the first calculation overestimated the loads.

The comparison is then extended to the characteristic numbers $\bar{A}$ and N_{os} of the responses, and shows that both these numbers had also been overestimated in the first calculation. The influence on fatigue life is then studied, assuming for simplicity that Miner's rule is valid, and that Wohler's law is a power law. The prediction of fatigue life is made for two critical points of the structure, considering that the only source of excitation is the atmospheric turbulence.

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The two predictions differ by a factor of the order of ten, and it is thought that they would differ by a factor of at least two when turbulence is considered only as part of a ground-to-air cycle.

Critical Analysis of the Assumption of Spanwise Uniformity

Setting of the Problem

When computing the response of an aircraft to turbulence, one of two assumptions is usually made (Fig. 1). According to the first one, the aircraft is assumed to fly through cylindrical random waves; according to the second one, the turbulent field is isotropic, i.e., no longer uniform in span. This first section is concerned with the limits of validity of these assumptions.

As a matter of fact, it can be concluded from the large number of spectra collected all around the world that atmospheric turbulence is not far from being isotropic, and is fairly well represented, in most cases, by Bullen spectral model. According to these assumptions, and using Taylor's hypothesis, one can express the cross correlation of the vertical component of the turbulent velocity between two points $M(x,y)$ and $M'(x',y')$ of a lifting surface (Fig. 2) flying at mean speed V_o, by

$$R_w(M,M',\tau) = g(\{[V_o\tau - (x-x')]^2 + (y-y')^2\}^{1/2}) \quad (1)$$

where $g(r)$ is the transverse correlation function of the turbulence field.

Fourier Transform yields the cross power spectrum

$$S_w(M,M',\omega) = G(\omega)C(\omega;\,|y-y'|/V_o)\exp[-i\omega\,(x-x')/V_o] \quad (1b)$$

where $G(\omega)$ is Bullen's[9] transverse spectrum

$$G(\omega) = (L\sigma_w{}^2/\pi V_o)[1 + 2\omega^2k^2(p+1)]/(1+\omega^2k^2)^{p+3/2} \quad (2)$$

This spectrum agrees reasonably well with most of the experimental results. It corresponds to Von Kármán's spectrum for $p = \frac{1}{3}$ and to Dryden's model for $p = \frac{1}{2}$. In Eq. (2), k has the dimension of a time, related to the scale of turbulence L by

$$k = L\Gamma(p)/V_o\pi^{1/2}\Gamma(p+\tfrac{1}{2})$$

In Eq. (1b), where one has written

$$\eta = |y-y'|/V_o$$

$C(\omega;\eta)$ is the spanwise coherence function which is expressed as

$$C(\omega;\eta) = 1 - \frac{|\eta|}{G(\omega)}\int_0^{+\infty} G((\omega^2+v^2)^{1/2})J_1(|\eta|v)dv \quad (3)$$

As it appears from Eq. (1b) that two-dimensional effects appear only because of the presence of $C(\omega;\eta)$, this function will be used to decide whether the assumption of uniformity of turbulence along span is valid or not.

Calculation of the Coherence Function

By use of Nielsen's integral

$$\int_0^{+\infty}\frac{\alpha^{\lambda+1}J_\lambda(\alpha x)d\alpha}{(\alpha^2+y^2)^{\mu+1}} = \left(\frac{x}{2y}\right)^\mu \frac{y^\lambda}{\Gamma(\mu+1)}K_{\lambda-\mu}(xy)$$

extended to the limit $\lambda = -1$, one can compute $C(\omega;\eta)$ as

$$C(\omega;\eta) = [2(p+\tfrac{3}{2})/\Gamma(p+\tfrac{1}{2})](X/2)^{p+3/2}[K_{p+3/2}(X) - AK_{p-1/2}(X)] \quad (4)$$

where

$$X = (\eta/k)(1+k^2\omega^2)^{1/2}$$

and

$$A = 2(p+1)(1+k^2\omega^2)/[1+2k^2\omega^2(p+1)]$$

In nearly all practical applications, the wavelengths associated with the modes fall in the inertial subrange, which means that

$$(L^2\omega^2/\pi V_o{}^2)(\Gamma(p)/\Gamma(p+\tfrac{1}{2}))^2 \gg 1$$

and yields the following simplification

$$(1+k^2\omega^2)^{1/2} \approx k\omega$$

Under this condition, Eq. 4 takes the very simple form

$$C(\omega;\eta) = [2(p+\tfrac{3}{2})/\Gamma(p+\tfrac{1}{2})](\omega\eta/2)^{p+3/2}\times [K_{p+3/2}(\omega\eta) - K_{p-1/2}(\omega\eta)]$$

or, with the use of Bessel functions recurrence formula

$$C(\omega;\eta) = [2/\Gamma(p+\tfrac{1}{2})](\omega\eta/2)^{p+1/2}K_{p+1/2}(\omega\eta) \quad (5)$$

This function depends only on the product

$$\xi = \omega\eta$$

For instance, for Dryden's model, one has

$$C(\xi) = \xi K_1(\xi)$$

and, for Von Kármán's model

$$C(\xi) = [2/\Gamma(\tfrac{5}{6})](\xi/2)^{5/6}K_{5/6}(\xi)$$

Figure 3 gives the graph of $C(\xi)$ for Dryden's model.

Transverse Coherence Length

The next step is to compute the transverse coherence length, defined as

$$l = \int_0^{+\infty} C\left(\omega\frac{y}{V_o}\right)dy$$

which can be done by use of the known integral formula

$$\int_0^{+\infty}\alpha^\mu K_v(\alpha)d\alpha = 2^{\mu-1}\Gamma\left(\frac{\mu+v+1}{2}\right)\Gamma\left(\frac{\mu-v+1}{2}\right)$$

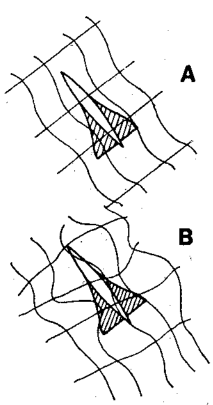

Fig. 1 Sketch of the turbulent field a) spanwise uniform, and b) isotropic.

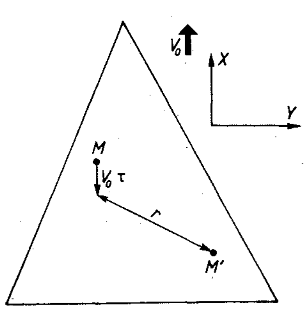

Fig. 2 Propagation of information on a wing.

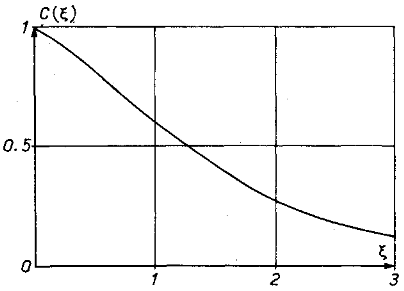

Fig. 3 Transverse coherence function for Dryden's model.

One finally obtains

$$l = (V_o/\omega)\Gamma(\tfrac{1}{2})\Gamma(p+1)/\Gamma(p+\tfrac{1}{2}) \tag{6}$$

For the values of p which are characteristic of actual atmospheric turbulence, i.e., $p = \frac{1}{3}$ to $p = \frac{1}{2}$, one obtains

$$1.40 < \Gamma(\tfrac{1}{2})\Gamma(p+1)/\Gamma(p+\tfrac{1}{2}) < 1.57$$

and, in first approximation

$$l \approx 1.43(V_o/\omega)$$

As has been explained previously, if the turbulence is nearly constant in span, the transverse coherence function will be close to unity, and the transverse coherence length will be much greater than the spanwise dimension.

Whenever the length of coherence l (at velocity V_o and for a natural mode of angular frequence ω) is much greater than the span dimension b, the assumption of cylindrical turbulent waves will be of practical use and will not lead to unacceptable errors. On the contrary, once

$$u = l/b = 1.43\ V_o/b\omega \tag{7}$$

is of the order of unity, or less, it will be necessary to perform the computation taking into account the isotropy of the turbulence pattern. Equation (7), which can be worked out quite easily with a slide rule, gives the required criterion of validity of the assumption of spanwise uniform turbulence, "Only when $u \gg 1$ can spanwise uniformity be assumed."

It is interesting to observe from Eq. (6) that the coherence length l does not depend on the scale of turbulence L. This result is consistent with the fact, by now generally recognized, that for most airplanes, the value selected for L does not affect the gust loads obtained, as long as the value of the rms gust velocity is properly adjusted for consistency with the value of L assumed.

When applied to practical examples, Eq. (7) shows that the transfer function, when limited to the range of frequency of rigid-airplane response, can be computed straightforward assuming that turbulence is cylindrical. On the other hand, more often than not, where flexible modes are concerned, the isotropy concept must be used. Table 1 summarizes these results, giving the values of u for four planes flying in cruise conditions.

Table 1 Values of u in cruise conditions

Aircraft	Pitch mode	First bending mode
Caravelle	4	1
B 707	5	0.8
Concorde	10	1.3
B 747	7	1.1

Computation of Transfer Function to Isotropic Turbulence

Mathematical Approach

What has been demonstrated in the previous section makes it necessary to develop a method for computing the transfer function of a flexible aircraft to isotropic turbulence. ONERA has developed such a method which is quite easy to handle and which is based on the techniques of the lifting surface theory.

Starting with the modal representation of the vertical displacement of the point M of the structure, considered as a linear superposition of the movements associated to the natural modes

$$z(M,t) = \textstyle\sum_k q_k(t)\psi_k(M)$$

one can express the power spectral density of the displacements induced by turbulence by

$$\phi_{zz}(M,\omega) = \sum_{k,p,r,s} \psi_k(M)\psi_p(M)T_{ps}{}^*(\omega)T_{kr}(\omega)B_{rs}(\omega)$$

where $T_{ij}(\omega)$ is the transfer function of mode i to excitation on mode j in flight conditions; $B_{rs}(\omega)$ is the cross power of the generalized forces induced by turbulence on modes r and s

$$B_{rs}(\omega) = \iint_A \iint_A S_p(\mu,\mu',\omega)\psi_r(\mu)\psi_s(\mu')d\sigma d\sigma' \tag{8}$$

In Eq. (8), $S_p(\mu,\mu',\omega)$ is the cross power of the pressure field induced by turbulence. Once this quantity is known, the problem is solved.

We shall use the integral equation of the lifting surface theory

$$\alpha(M,\omega) = \iint_A K(M,\mu,\omega)p(\mu,\omega)d\sigma \tag{9}$$

which relates the local pressure $p(\mu,\omega)$ to the local angle of attack $\alpha(M,\omega)$ at a given frequency ω. Different techniques can be used to solve the problem; in any event, one obtains the solution as the matrix equation

$$\{C_p\} = \{G\}\{\alpha\}$$

where $\{\alpha\}$ is the column matrix of the values of the angle of attack at N locations on the wing, $\{C_p\}$ the column matrix of the pressure at N points, and $\{G\}$ a square matrix which defines the influence coefficients.

After inverse Fourier Transform, and with evident notations Eq. (9) becomes

$$\tilde{\alpha}(M,t) = \int_{-\infty}^{+\infty} \iint_A \tilde{K}(M,\mu,\lambda)\tilde{p}(\mu,t-\lambda)d\lambda\ d\sigma$$

If the angle of attack is only due to the vertical component of turbulence $\tilde{W}(M,t)$, this equation takes the form

$$\tilde{W}(M,t) = V_o \int_{-\infty}^{+\infty} \iint_A \tilde{K}(M,\mu,\lambda)\tilde{p}(\mu,t-\lambda)d\lambda\ d\sigma \tag{10}$$

At point M', at time $t+\tau$, one should have

$$\tilde{W}(M',t+\tau) = V_o \int_{-\infty}^{+\infty} \iint_A \tilde{K}(M',\mu',\lambda')\tilde{p}(\mu',t+\tau-\lambda')d\lambda'\ d\sigma' \tag{11}$$

Averaging the product $\tilde{W}(M,t)\tilde{W}(M',t+\tau)$, assuming the process is stationary and noting by $*$ the time convolution, one can express the cross correlation $R_w(M,M',\tau)$ of the turbulence as a function of the cross correlation $\tilde{S}_p(M,M',\tau)$ of the pressure field

$$R_w(M,M',\tau) = V_o^2 \iint_A \iint_A \tilde{K}(M,\mu,-\tau) * \tilde{K}(M',\mu',\tau) * \tilde{S}_p(\mu,\mu',\tau)\, d\sigma\, d\sigma' \tag{12}$$

which yields, after direct Fourier Transform

$$S_w(M,M',\omega) = V_o^2 \iint_A \iint_A K^*(M,\mu,\omega) K(M',\mu',\omega) S_p(\mu,\mu',\omega) d\sigma\, d\sigma' \tag{13}$$

Eq. (13) is an integral equation which relates the required cross power of the pressure field to the known cross power of the vertical component of turbulence. As the kernel K^*K is factorized, the inversion of this equation is by no means more complicated than in the simple case of the lifting surface theory. One obtains the solution in matrix form

$$\{S_p\} = (1/V_o^2)\{G^*\}\{S_w\}\{G\} \tag{14}$$

where $\{S_w\}$ is the square matrix representing the cross power of the turbulent field (evaluated from Eq. (1b) for N^2 relative positions), $\{S_p\}$ the square matrix of the pressure field at N^2 locations on the wing, and $\{G\}$ the aerodynamic influence matrix that has always been previously computed for the purpose of flutter prediction. One can see from Eq. (14) that the only small complication coming from the concept of isotropy lies in the evaluation of $\{S_w\}$ from Eq. (1b).

Practical Application

The method has been applied for predicting the response in acceleration at four different locations on the structure of the Anglo-French SST; at Mach numbers 0.4 and 0.8. Some of these results are given in Fig. 4, with comparison with previous calculations that were made assuming that turbulence was spanwise uniform.

The following important results should be noticed:

a) The vertical line drawn on all the graphs that corresponds to $u = 1$ gives a pretty good definition of the limit of validity of the assumption of uniformity. This supports the use of the criterion proposed in the previous section.

b) The spectra of the responses to turbulence, when computed with the assumption of isotropy, are much smoother, at high frequencies, than the spectra computed by the former method. This trend is consistent with the results of comparisons of computed transfer functions and those measured in flight.

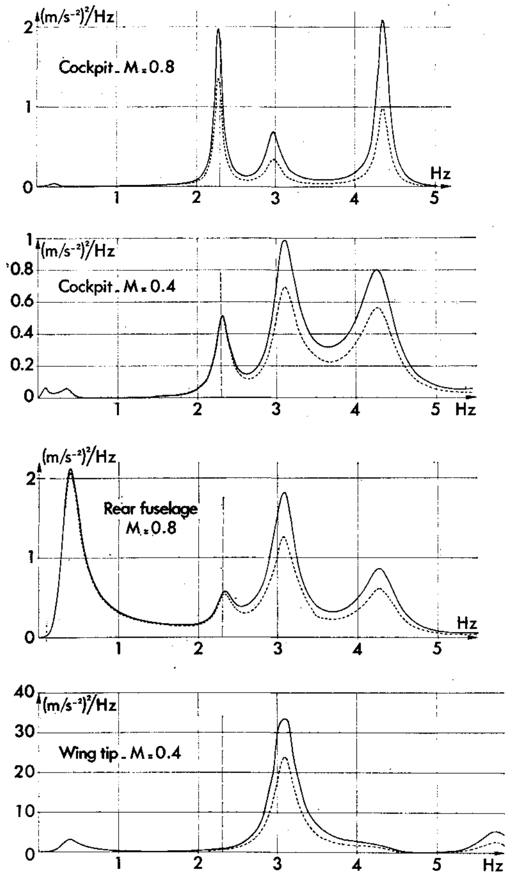

Fig. 4 Power spectral density of the vertical acceleration; uniform turbulence, Isotropic turbulence.

Influence on the Prediction of Fatigue Life

Load Statistics

For the purpose of predicting the fatigue life of an aircraft, the first step is to choose a model of atmosphere and the second is to decide about a mission profile. Some cumulative damage rule is then used to predict, more or less accurately, the number of flights to failure; fatigue tests with load history simulation, are often carried out to support the prediction.

In the general harmonic analysis approach, the model of turbulence is the one proposed by Press, who considers the atmosphere as a set of patches of Gaussian stationary turbulence. All the patches have the same reduced power spectral density; they only differ by their respective rms σ_w. The description is completed by assuming a probability density $\Delta(\sigma_w)$ for σ_w.

The response of the aircraft (for instance the stress s at a given location) is also, for each patch, a Gaussian process, characterized by its rms

$$\sigma_s = \bar{A}\sigma_w$$

and its mean number of zero crossings

$$N_{os} = \frac{1}{2\pi\sigma_s}\left[\int_0^{+\infty} \omega^2 G(\omega)|T(\omega)|^2 d\omega\right]^{1/2} \tag{15}$$

where $T(\omega)$ stands for the transfer function of parameter s to turbulence.

For a patch characterized by rms σ_w, the mean number of times per second that stress exceeds a value s is given by Rice's[10] formula

$$n(\sigma_w,s) = N_{os}\exp(-s^2/2\bar{A}^2\sigma_w^2)$$

Consequently, the mean number of crossings per second of a level of the stress will be expressed, for the over-all process, by

$$n(s) = N_{os}\int_0^{+\infty} \Delta(\sigma_w)\exp\left(-\frac{s^2}{2\bar{A}^2\sigma_w^2}\right) d\sigma_w \tag{16}$$

It appears from Eq. (16) that the statistical distribution of the stress will depend on the values of $\bar{A}$ and N_{os}, which depend themselves on the way the calculation of the transfer function has been carried out.

$\bar{A}$ and N_{os} relative to stress are not available for the Anglo-French SST. Nevertheless, these numbers have been calculated for the vertical acceleration at some locations on the structure. It is thought that these results, summarized in Table 2, will give the main trends of the phenomenon.

The discrepancies between the two predictions are of the order of 14% for $\bar{A}$ and of 20% for N_{os}.

Sensitivity of Fatigue Life to the Values of $\bar{A}$ and N_{os}

For the purpose of simplicity, the influence of $\bar{A}$ and N_{os} on the fatigue life of the structure will be evaluated assuming that Miner's law for cumulative damage is valid, and that the

Table 2 Comparison of $\bar{A}$ ($\sec^{-1}$) (upper number) and N_{os} (lower number)[a]

Location	Cylindrical turbulence[b]	Isotropic turbulence[b]
Cockpit	1.42 5.18	1.22 4.37
Center of Gravity	1.62 5.26	1.43 4.44
Wing tip	5.79 18.12	4.87 14.99
Rear	2.15	1.94
fuselage	6.02	5.09

[a] Computed with cylindrical or isotropic turbulence concorde, M = 0.8.
[b] Effect on numbers $\bar{A}$ and N_{os}.

(s,N_r) Wholer's law is a power law. This means that the number $N_r(s)$ of cycles to failure under stress s is given by

$$N_r(s) = (s_o/s)^{2p}$$

Once again for the purpose of simplicity, it will be assumed that turbulence is the only source of fatigue for the structure. For high enough values of a purely Gaussian process (say, twice the rms σ_s of the stress), the number of cycles $N_c(s)\delta s$ for which the stress amplitude lies between s and $s+\delta s$, in a sample of duration T, can be approximated by

$$N_c(s)\delta s = (TN_{os}s/\sigma_s^2)\exp(-s^2/2\sigma_s^2)\delta s$$

and the increase in damage, due to these cycles, is

$$\delta d = N_c(s)\delta s/N_r(s) = (TN_{os}/\sigma_s^2)(s/s_o)^{2p}s\exp(-s^2/2\sigma_s^2)\delta s$$

With the help of Miner's lure, the over-all damage for the Gaussian sample is deduced as being

$$d = TN_{os}\left(\frac{\sigma_s}{s_o}\right)^{2p}\int_0^{+\infty} v^{2p+1}\exp\left(-\frac{v^2}{2}\right)dv$$

where v is defined as s/σ_s. This last integration can be carried out without any difficulty and one finds

$$d = 2^p(p)!TN_{os}(\sigma_s/s_o)^{2p}$$

The over-all damage $\bar{d}$ induced by the complete set of Gaussian patches is then expressed by

$$\bar{d} = TCN_{os}2^p(p)!(\bar{A}/s_o)^{2p}$$

where

$$C = \int_0^{+\infty}\Delta(\sigma_w)\sigma_w^{2p}d\sigma_w$$

depends only on the atmospheric properties, and not at all on the specific values of $\bar{A}$ and N_{os}.

Failure occurs when the cumulative damage reaches unity, which means that the fatigue life T_f is given by

$$T_f = 1/2^p(p)!N_{os}(\bar{A}/s_o)^{2p}C \tag{17}$$

If we call T_f' the fatigue life, when computed with inaccurate values $\bar{A}'$ find N'_{os} of $\bar{A}$ and N_{os}, one obtains the following ratio of fatigue lives:

$$T_f/T_f' = (N_{os}'/N_{os})(\bar{A}'/\bar{A})^{2p} \tag{18}$$

It will be noticed from Eq. (17) that fatigue life due to turbulence can be expressed quite easily. The parameter C, which includes only atmospheric properties, is the same at all locations on the structure.

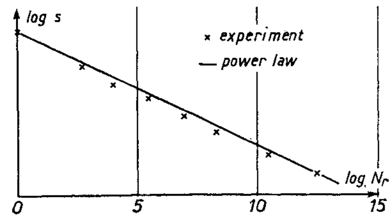

Fig. 5 Wohler's (s,Nr) curve for Concorde alloy.

Practical Application

Let us now try to apply Eq. (18) to compute the change in the predicted fatigue life of Concorde that occurs as a consequence of changing the model of turbulence from cylindrical to isotropic. For this purpose, it will be assumed that the error in $\bar{A}$ and N_{os} for stress is of the same order of magnitude as the corresponding error for acceleration.

In Fig. 5 is given the (s,N_r) Wohler's curve for the alloy used in Concorde. This curve is pretty well represented by the power law

$$N_r(s) = (s_o/s)^{12}$$

Coming back to Eq. (18) and assuming, as a consequence of the results shown in Table 2, that

$$\bar{A}'/\bar{A} = 1.14 \text{ and } N'_{os}/N_{os} = 1.20$$

one finds easily that

$$T_f = 5.80\ T_f'$$

The important conclusion that rises now is that the prediction of fatigue life due to turbulence can be changed by a factor of nearly 6 according to the assumption made on turbulence (uniform in span or isotropic). In fact, the increase is not so important, in view of the fact that turbulence is only part of the sources of loads in a ground-to-air cycle. Anyway, it can be thought that, for an aircraft like Concorde, the actual fatigue life, including all effects, would be changed by a factor of the order of 2 according to the assumption made on turbulence.

Conclusion

The comprehensive comparisons of the responses of large aircraft to turbulence, computed either with a model of turbulence uniform in span, or with a model of isotropic turbulence, exhibit severe discrepancies once flexible modes are concerned. The changes appear in the shape of the spectra as well as in the values of the characteristic numbers $\bar{A}$ and N_{os}. The effect of these discrepancies on the fatigue life of the structure may be very important when turbulence is one of the greatest factors of loading.

It is thus necessary to perform the calculation of the responses using an isotropic model of the turbulent field whenever the transverse coherence length is of the order of magnitude of the spanwise dimension of the aircraft. In this paper is proposed a very simple formula, which makes it possible to decide which model should be chosen in any particular case. All these results will be supported, in the next two years, by a great amount of flight data to be collected on Concorde preproduction aircraft.

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A General Class of Exact Airfoil Solutions

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Existing classical analytical airfoil solutions are not flexible enough to provide sufficiently critical test problems for modern numerical techniques. This paper presents the theory and some results for a very general and versatile form of conformal mapping which can be used to generate local analytical bumps and dips on airfoil profiles. The mapping uses an analytical modulating function constructed from multipoles within the unit circle, and various consequences of this format are presented—where they may have an influence on numerical test cases. For instance, the existence of curvature singularities at the trailing edges of most common airfoils (Joukowski, Von Mises, Karman-Trefftz, etc.) is made particularly evident. Results are presented for zero trailing edge angle from an IBM 2250 Graphics Display System, showing how a variety of severe test problems with exact solutions and involving local surface deformations can be generated at will.

Introduction

THE continual growth of well-founded modern numerical techniques has focused attention on details and accuracy considerations that were not important in the earlier days of their gross development. Furthermore, the widespread and general reduction of funding available to support ill-founded and wasteful schemes has generated timely interest in efficiency and maximum accuracy for a given computing time.

Many examples of this changing emphasis have occurred within the daily working life of engineers at Douglas, and provide a very clear case for the development of "exact" solutions adequate to investigate the detailed performance of numerical methods. For instance: a) the Douglas-Neumann[1] program is one of the major fully established tools for low-speed flow problems in the aircraft and associated industries. Even though the method is formally exact in two and three dimensions, the discretization used in practice leads very frequently to questions about the point spacing density required to extract sufficient flow detail in regions of rapid geometrical change. Typical practical cases are local lumps, thin trailing edges, close approach of slat trailing edges to main airfoils, very sharp noses, etc. b) Recent new methods[2,3] for airfoil problems where attention has been directed toward accuracy rather than generality of body shape (because of the subsequent intention to develop compressible versions) have intensified the need for adequate test problems. In the flow analysis part of Ref. 2, and in Ref. 3, the surface angle has to be extracted from coordinate data and its accuracy is an absolute limitation on convergence of the iterative cycle in both. For very sharp noses, or excessive camber, or flap hinge lumps—some guidance is needed on input detail and on the flow response to it. c) The design of high-lift airfoils by the methods of Ref. 2 according to the precepts of Ref. 4 have led to frequent discrepancies and futile speculations on accuracy between the various methods mentioned previously. These high-lift airfoils are very cambered, have sharp noses and unsmooth pressure distributions. Of course, the need is again to have some kind of "exact" solution which incorporates these features. d) Finally, there remains the possibly most important application of all, namely, guidance for the questions which arise from pure data-handling of geometry. For given coordinate data the production of "smoothed" input points, to say nothing of angles and curvature, remains a very real and difficult problem in spite of the abundance of numerical techniques. A number of point-spacing suggestions have been proposed for use with the Douglas-Neumann program and these usually involve arc length and curvature; which require in turn interpolation and differentiation of possibly noisy coordinate data. Hence, there is a natural concern with smoothing and associated numerical operations; but, again, it is effectively impossible to assess the various alternatives without being able to generate test cases. How many points are required to define a slat nose? How many if the data is noisy? and what should be their spacing? Such questions can only be approached (never mind answered) by starting with a sufficiently unpleasant geometry and adding

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